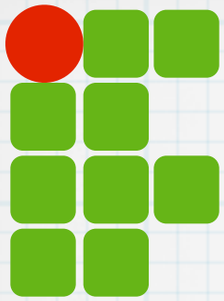
INSTITUTO FEDERAL DE
EDUCAÇÃO, CIÊNCIA E TECNOLOGIA
RIO GRANDE DO NORTE

Algoritmos

Programação dinâmica



Agenda

* Programação dinâmica
* Exemplos
* Exercícios

O que é?

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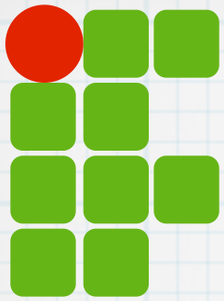
* Problemas resolúveis recursivamente (Combinação de soluções de subproblemas)



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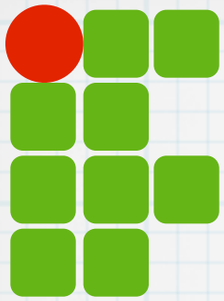

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2) Economizar memória, resolvendo sub-problemas por ordem que minimiza nº de soluções parciais a memorizar (bottom-up, começando pelos casos base)

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* Termo "Programação" vem da Pesquisa Operacional, no sentido de **formular restrições ao problema que tornam um método aplicável**

Exemplo

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$$C_k^n = \frac{n!}{k! \times (n-k)!}$$

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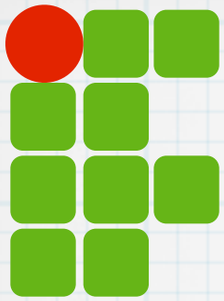

Exemplo

```c
int comb(int n, int k) {
   if (k == 1)
      return n;
   if (n == k)
      return 1;
   return (comb(n - 1, k - 1) + comb(n, k - 1));
}
```

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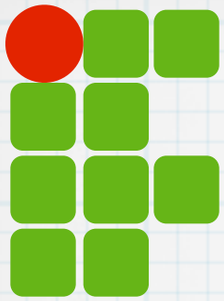

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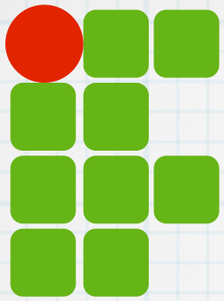

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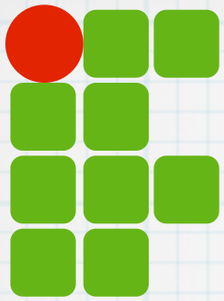

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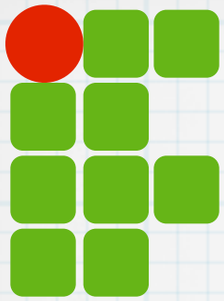
Comb(5,2)



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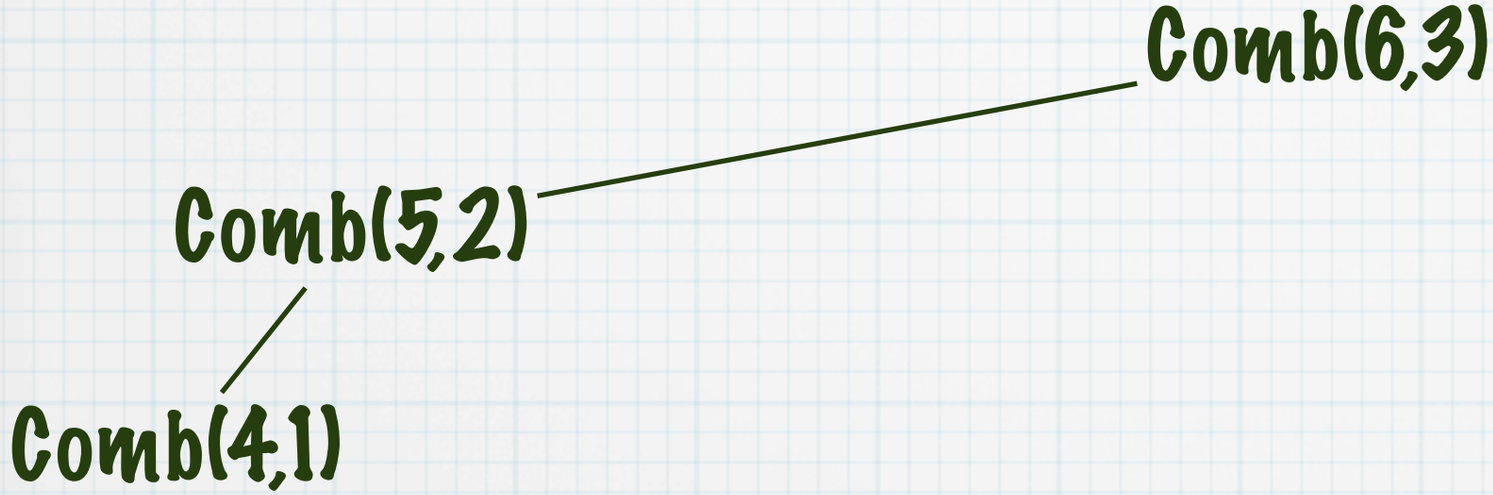

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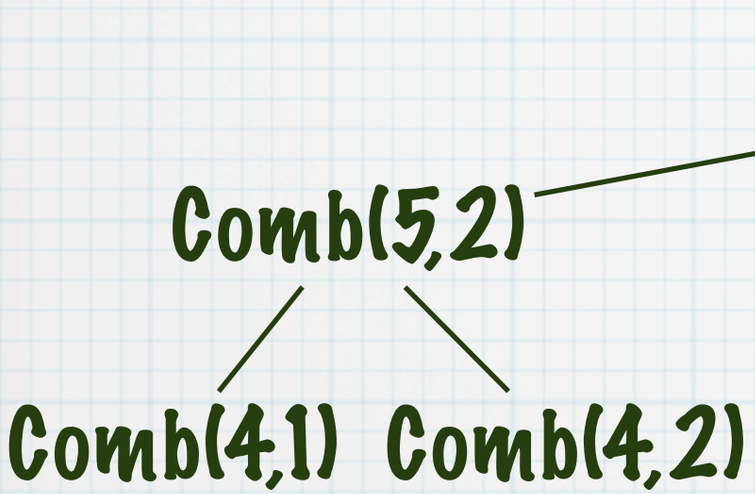

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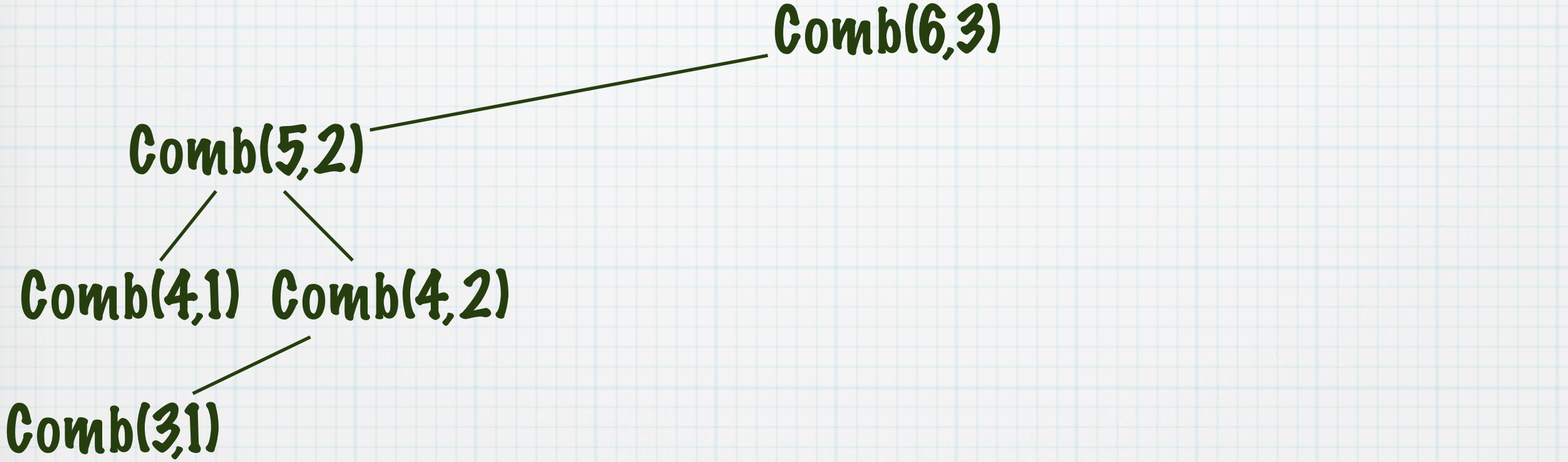

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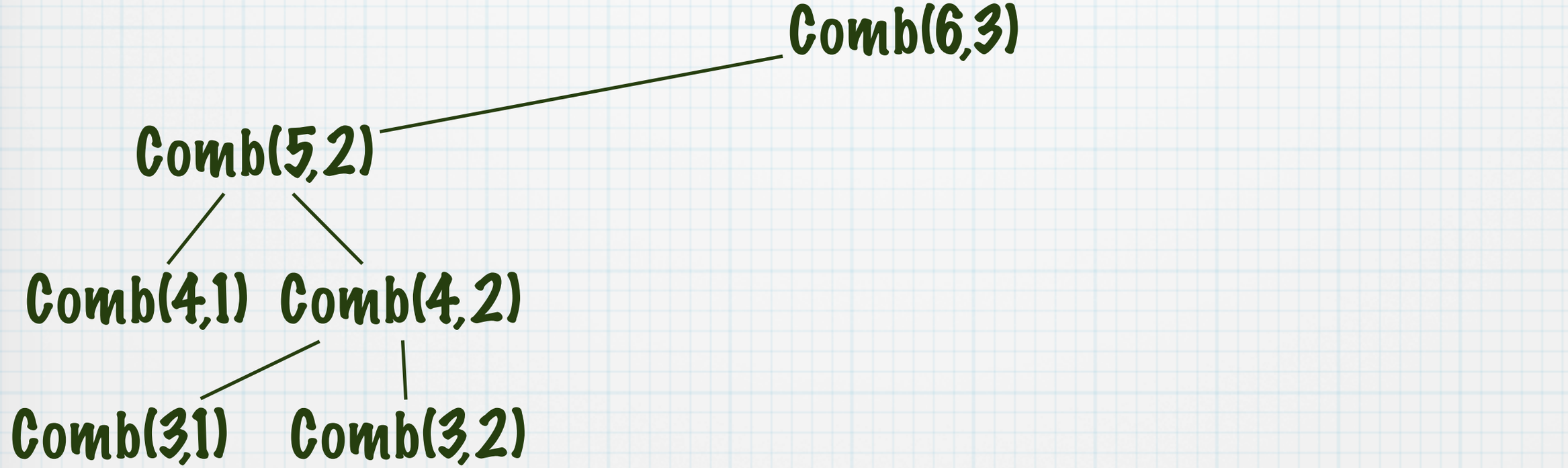

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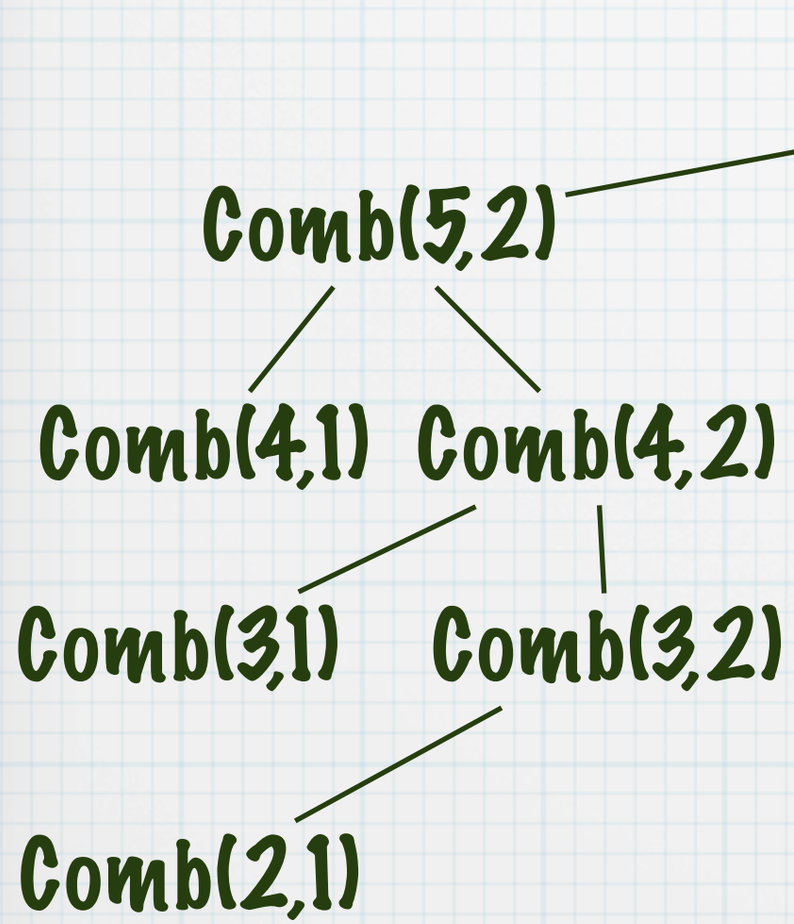

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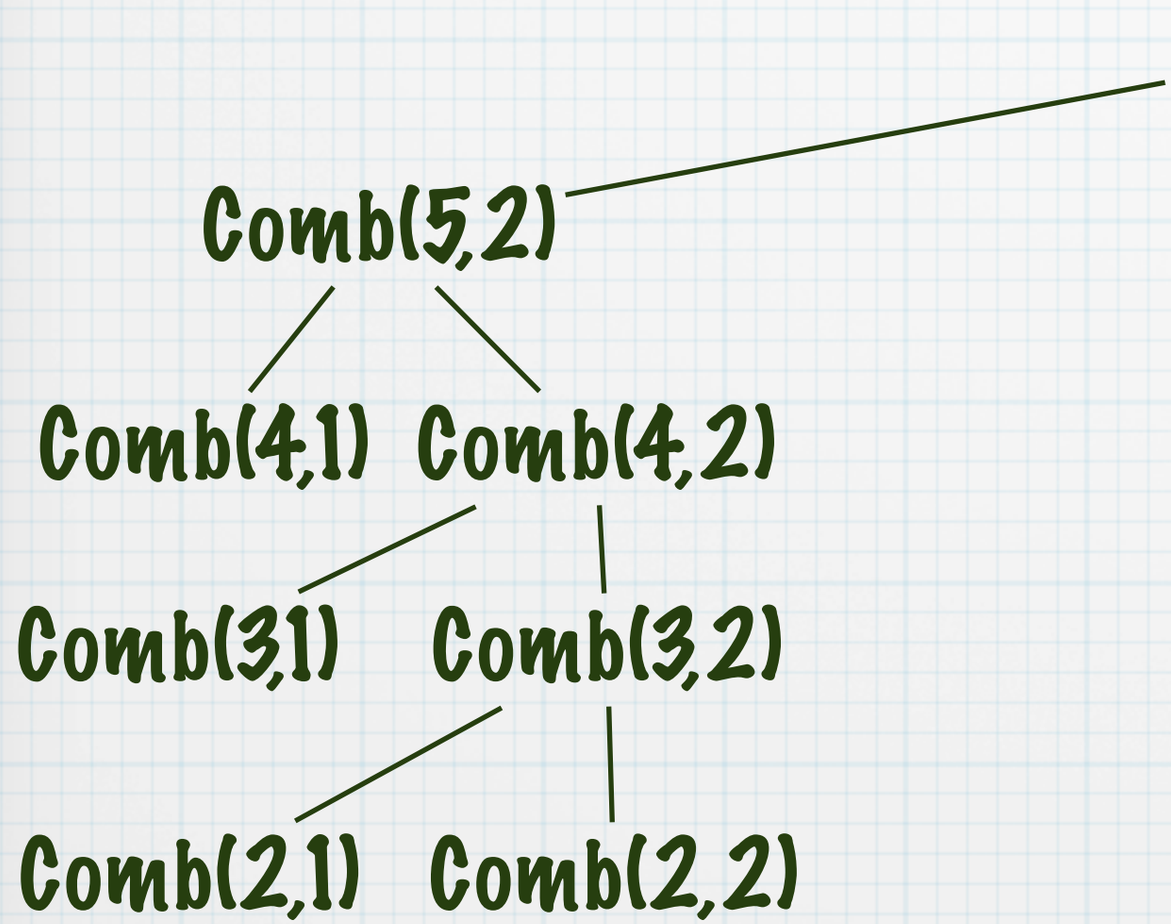

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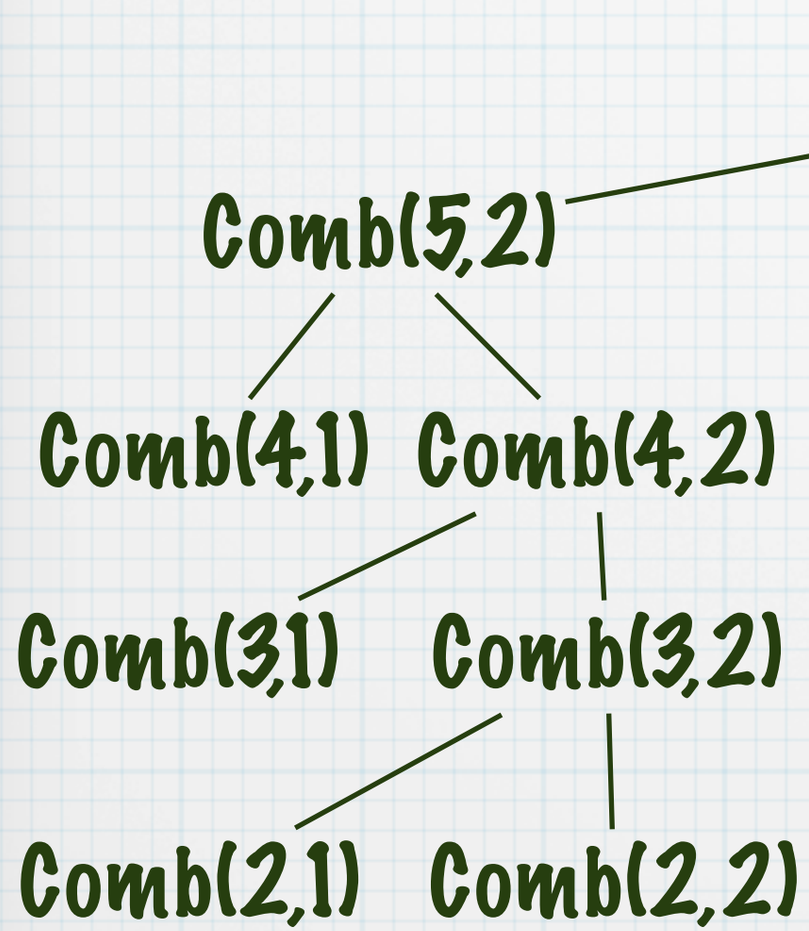

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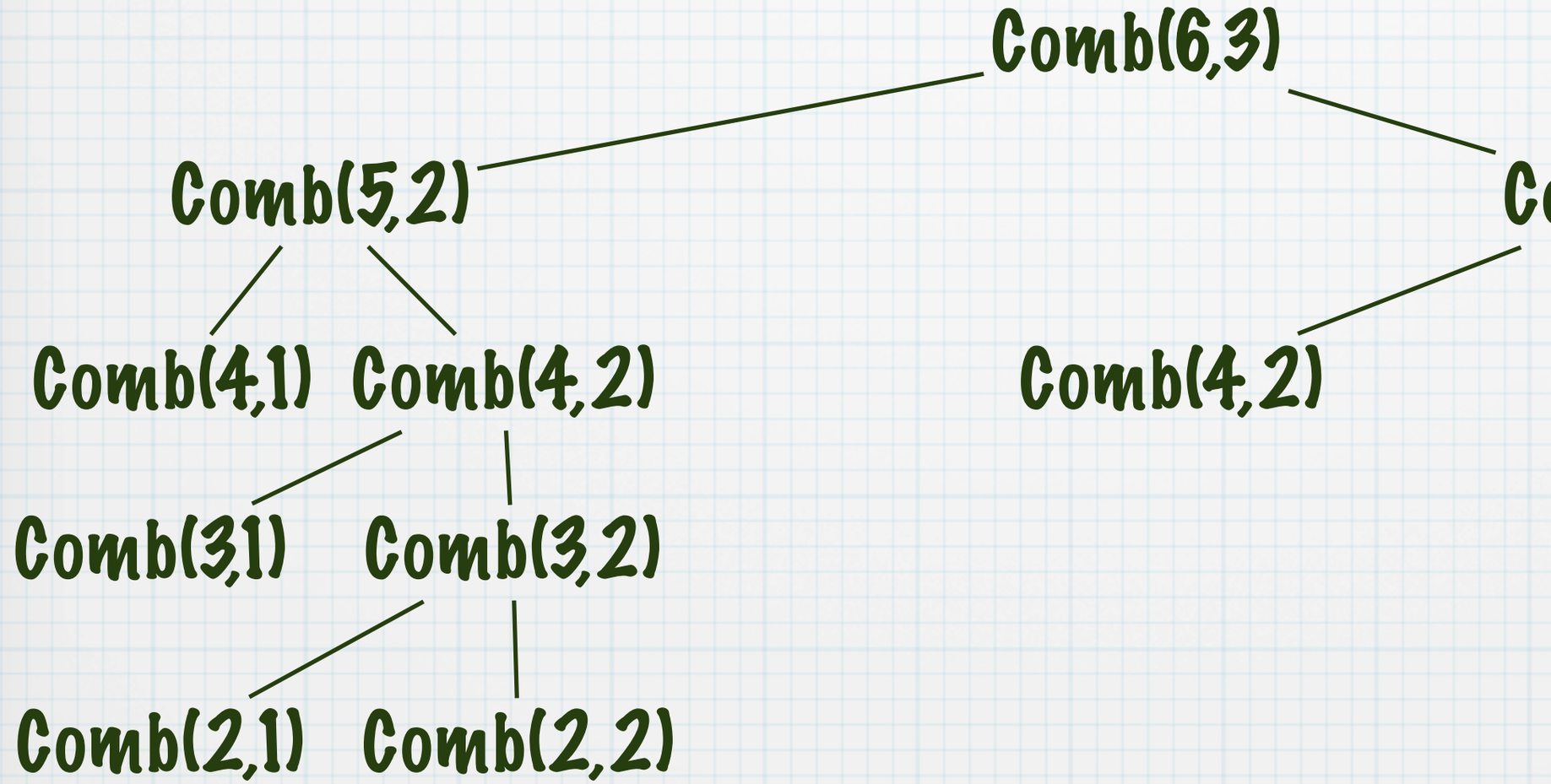

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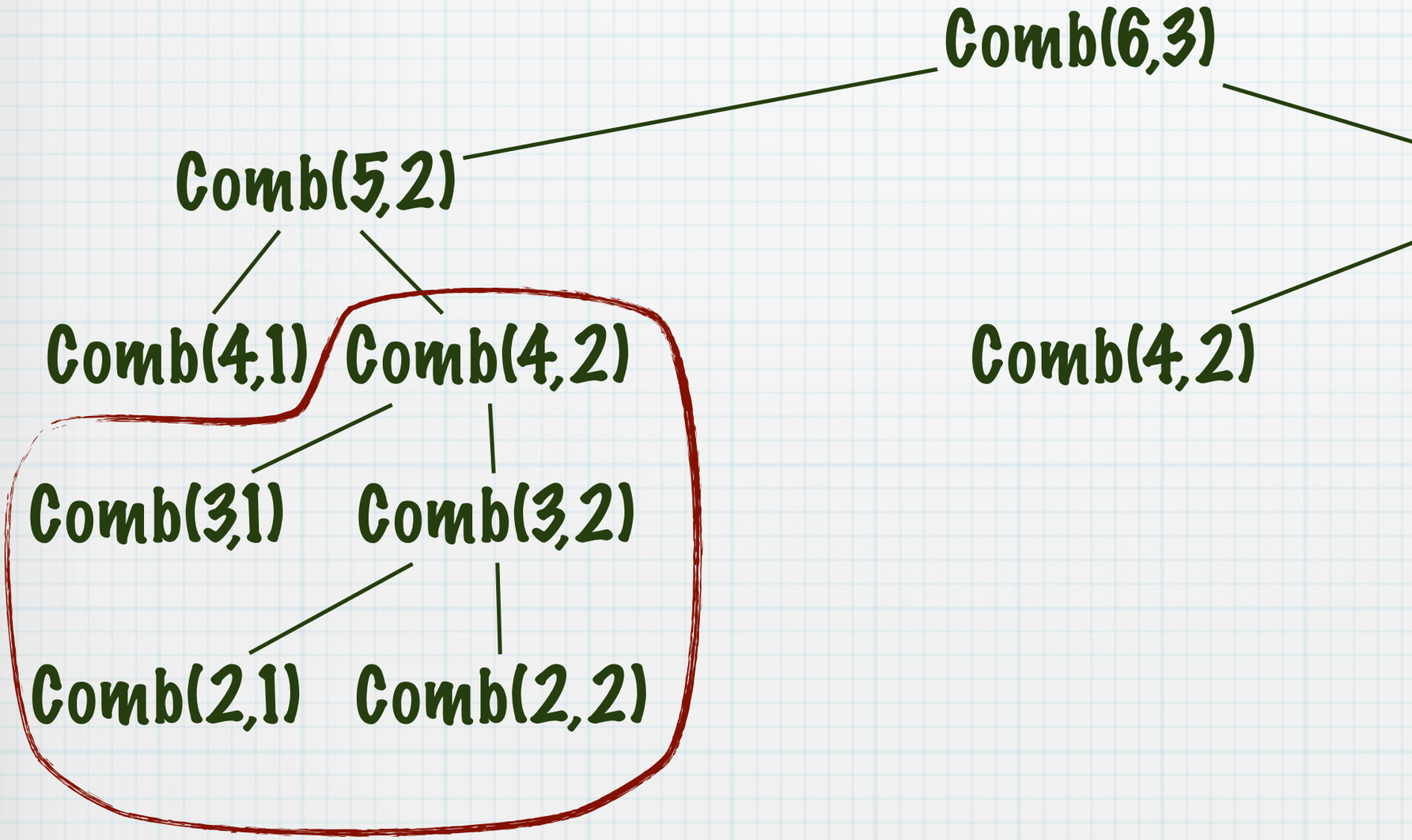

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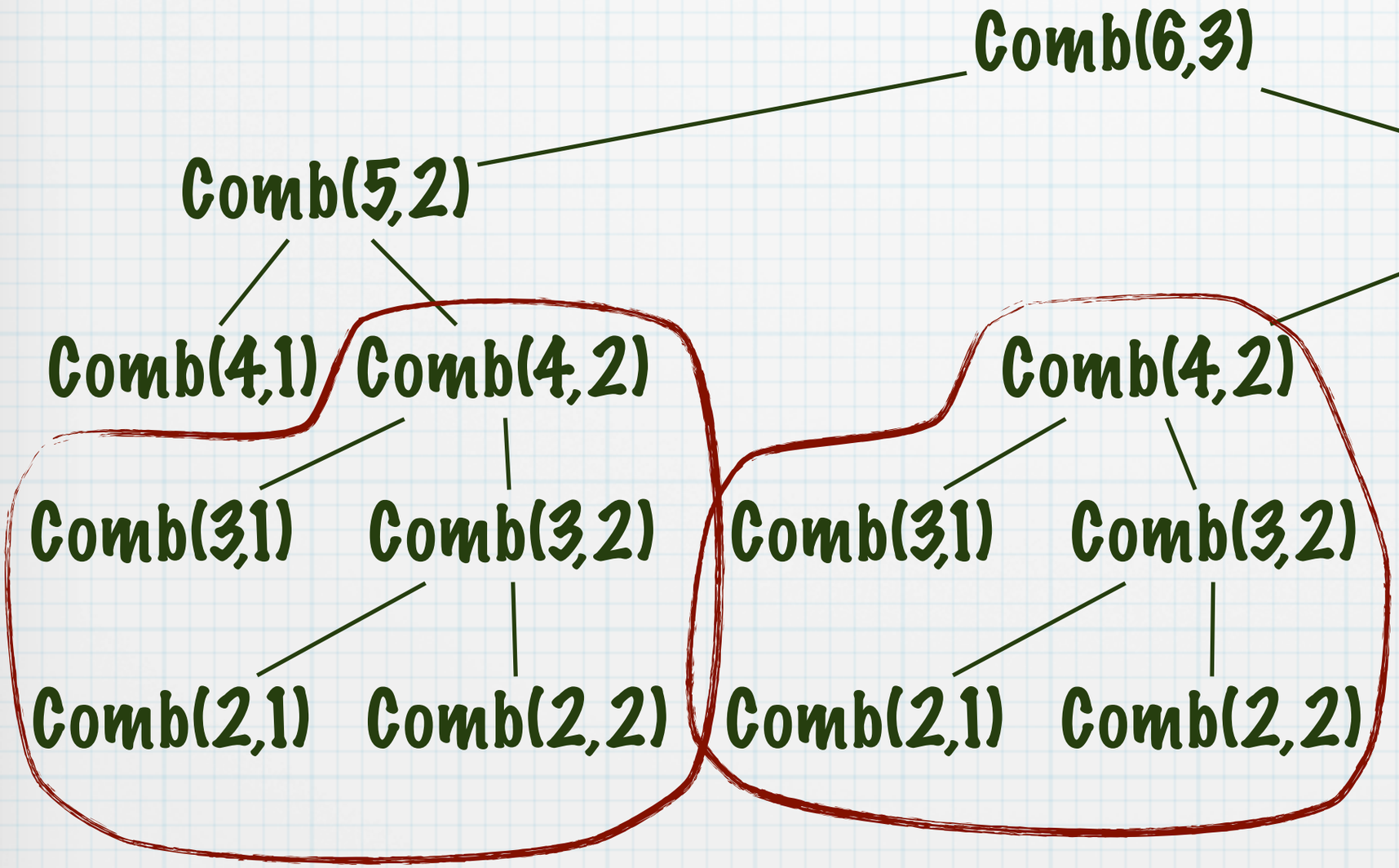

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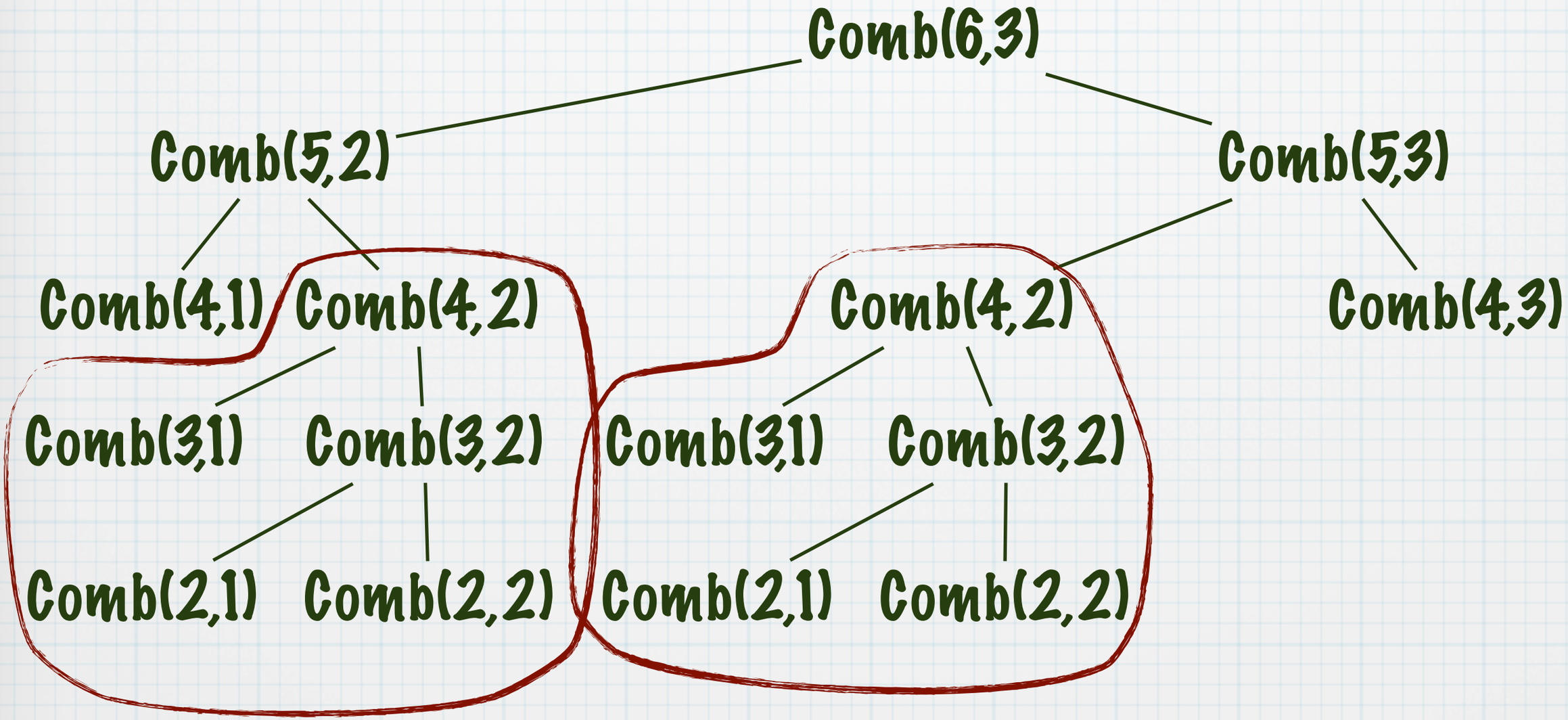

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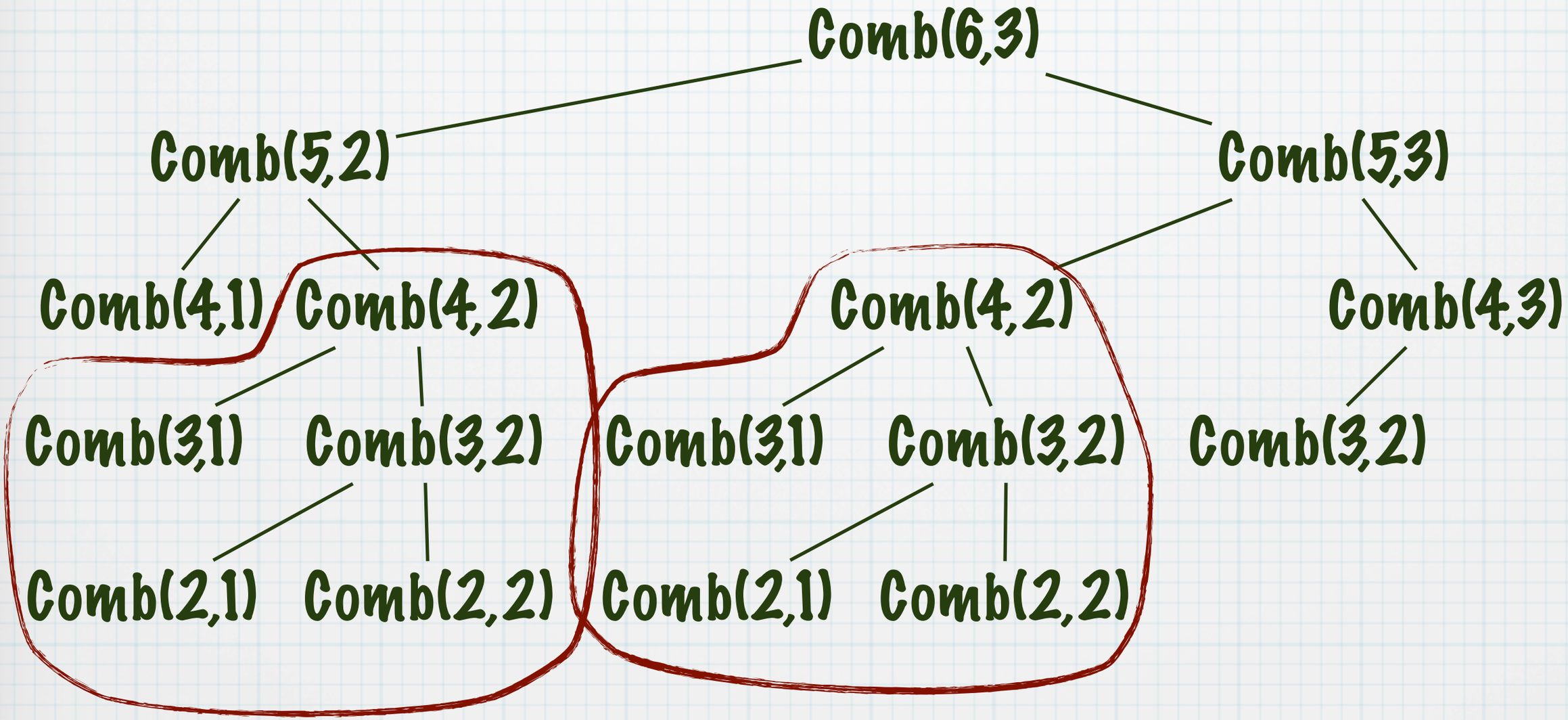

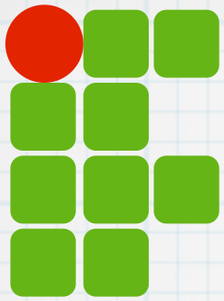

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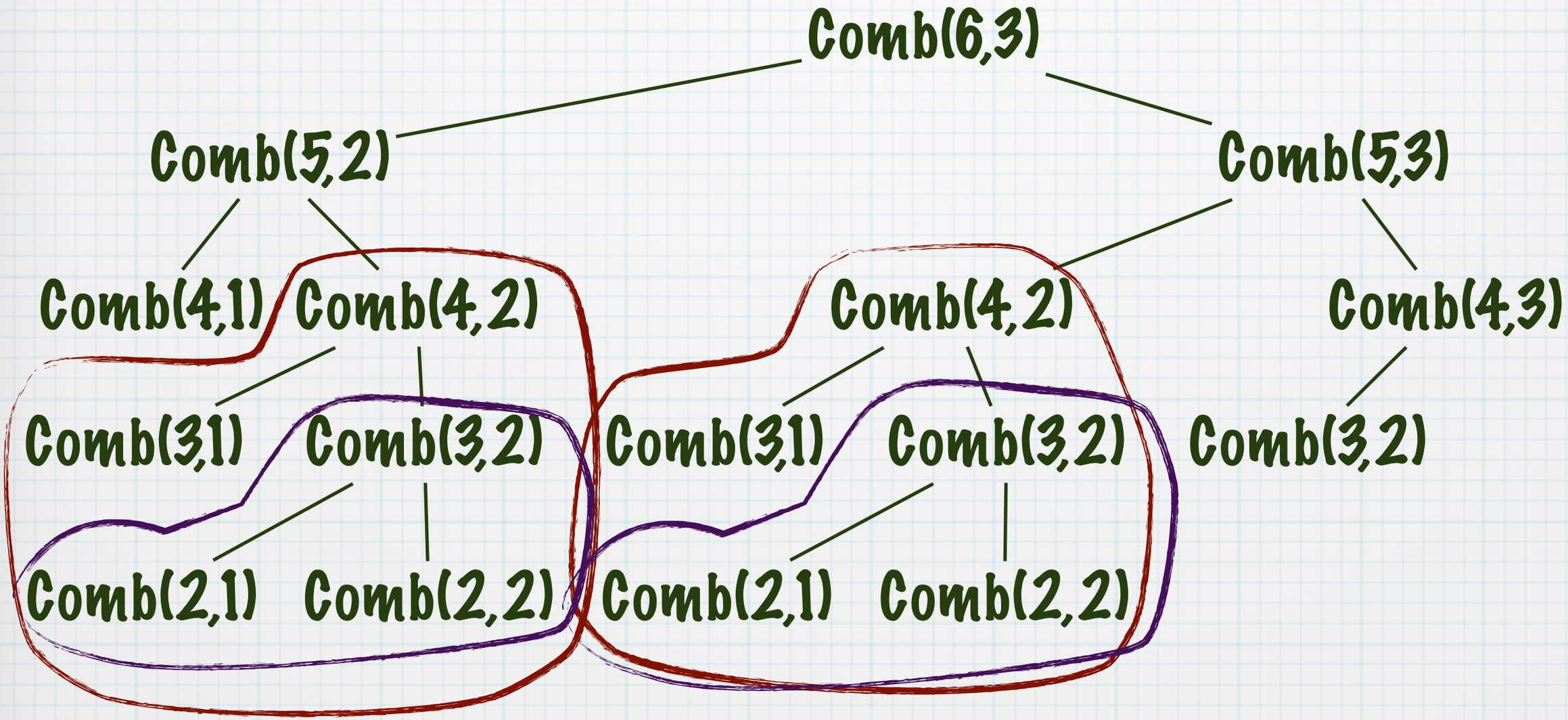

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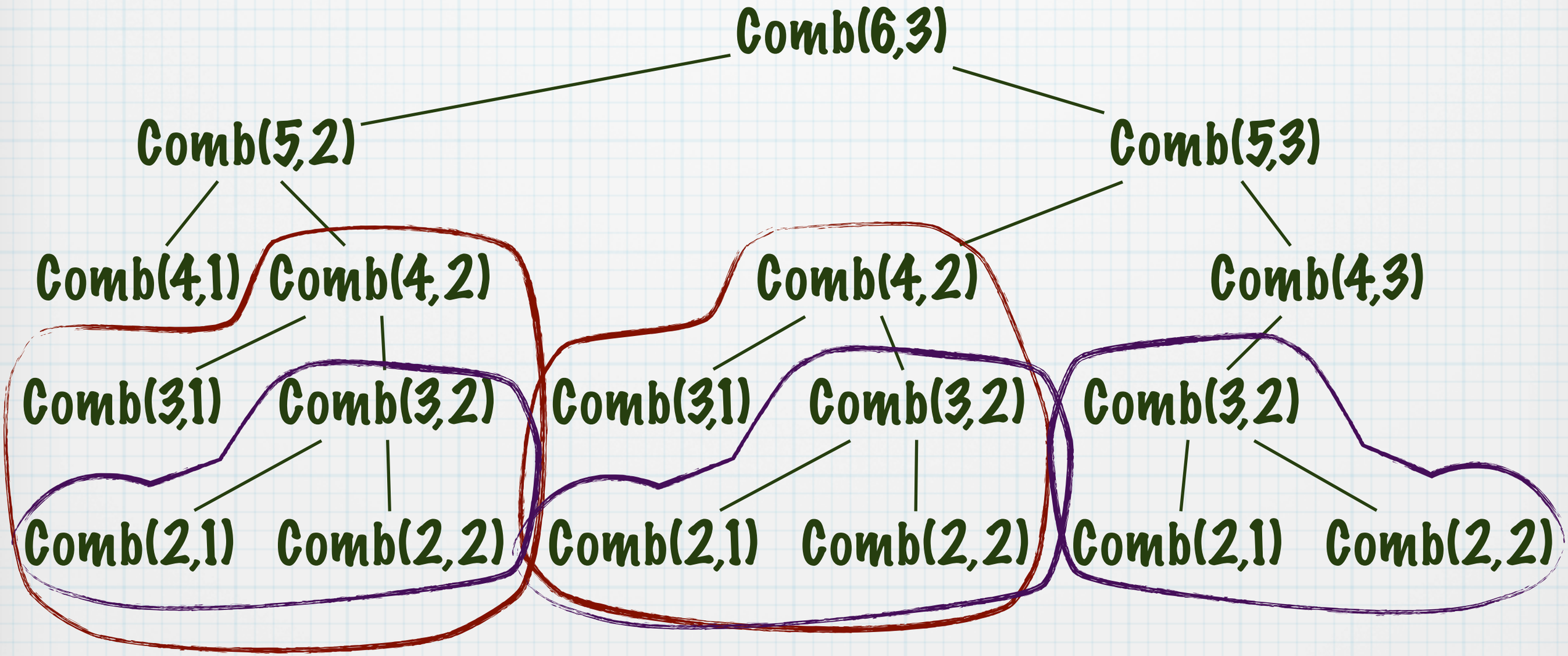

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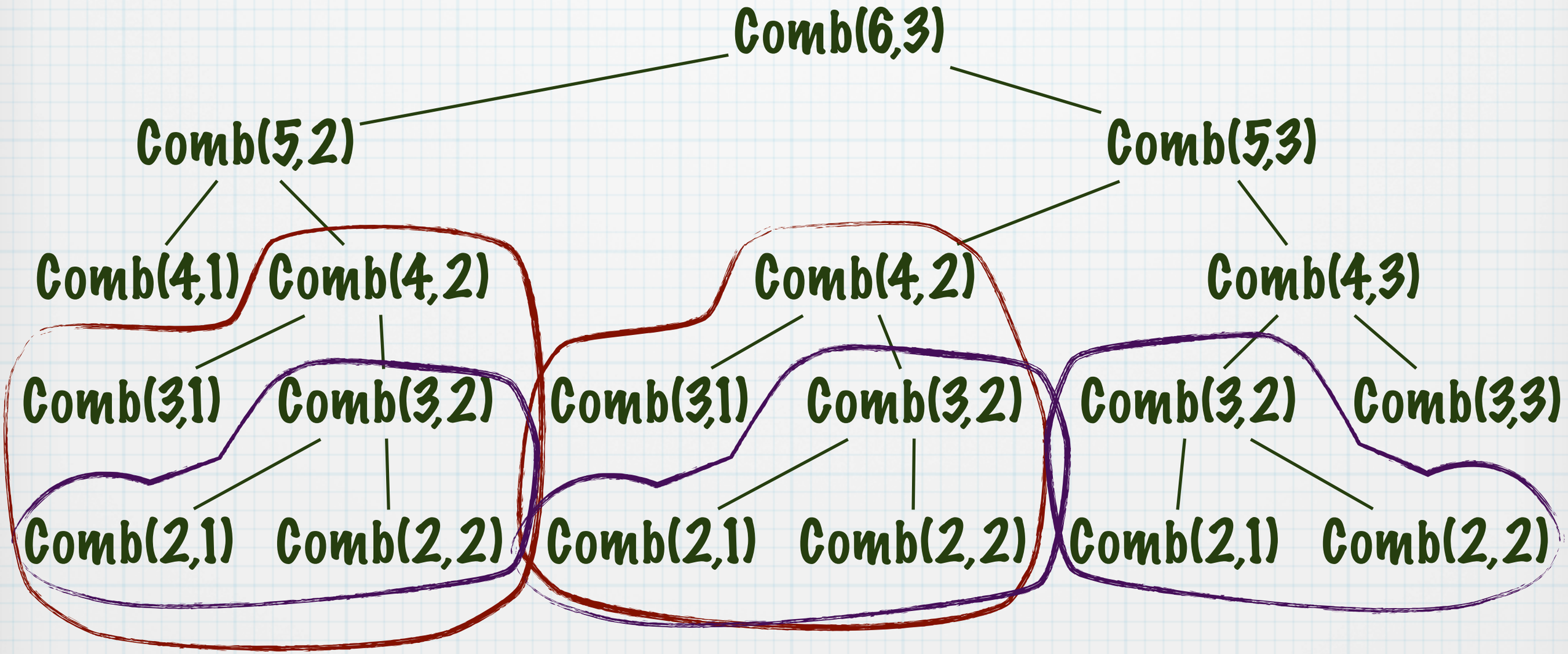

Exemplo

C_k^n	k=1	k=2	k=3	k=4
n=1	1			
n=2	2	1		
n=3	3	3	1	
n=4	4	6	4	1
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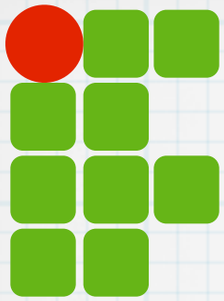

Exemplo

```
int comb(int n, int k) {
   int i, j;
   int maxj = n - k;
   int *c = malloc(sizeof(int) * maxj+1);
   int result = 0;
   for (j = 0; j <= maxj; j++)
      c[j] = j + 1;
   for (i = 1; i < k; i++) {
      for (j = 1; j <= maxj; j++)
         c[j] += c[j - 1];
   }
   result = c[maxj];
   free(c);
   return result;
}
```

Exemplo

1
2
3
4

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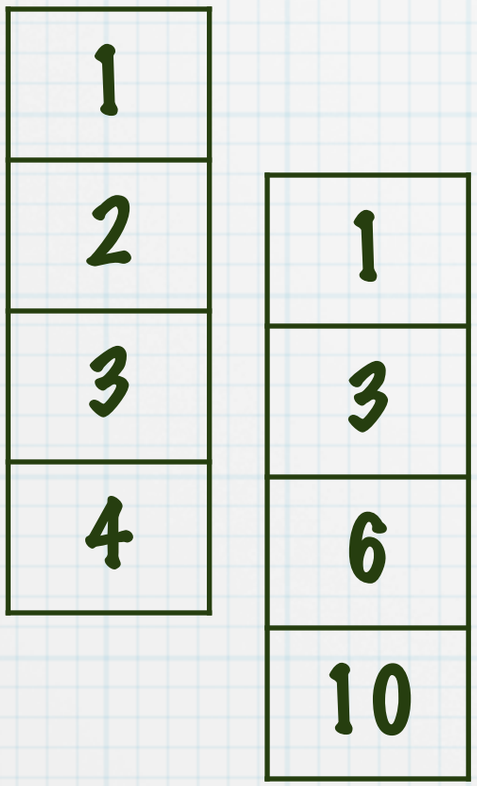


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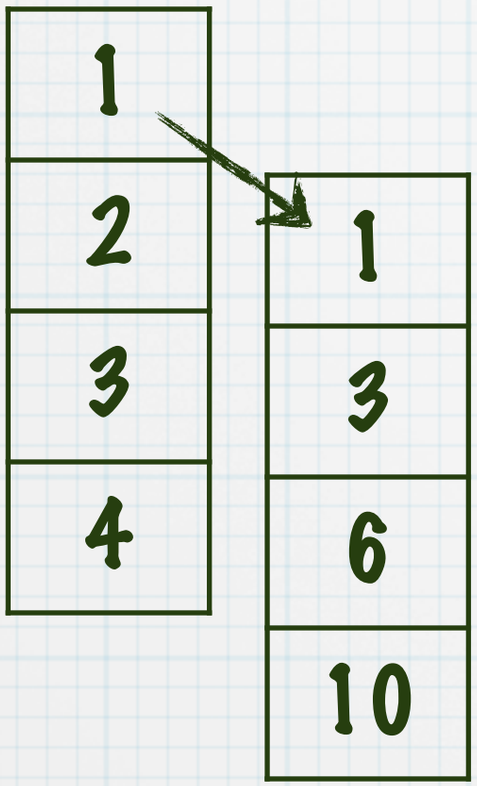
   int result = 0;
   for (j = 0; j <= maxj; j++)
      c[j] = j + 1;
   for (i = 1; i < k; i++) {
      for (j = 1; j <= maxj; j++)
         c[j] += c[j - 1];
   }
   result = c[maxj];
   free(c);
   return result;
}
```

i=1

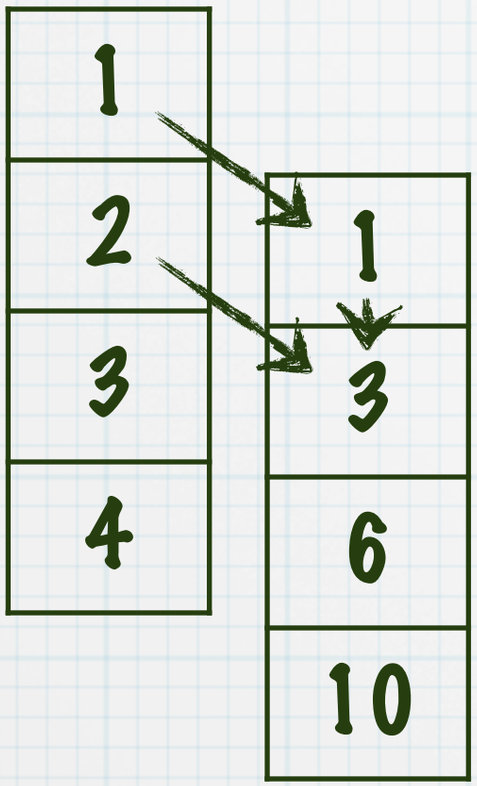
Exemplo




```
int comb(int n, int k) {
   int i, j;
   int maxj = n - k;
   int *c = malloc(sizeof(int) * maxj+1);
   int result = 0;
   for (j = 0; j <= maxj; j++)
      c[j] = j + 1;
   for (i = 1; i < k; i++) {
      for (j = 1; j <= maxj; j++)
         c[j] += c[j - 1];
   }
   result = c[maxj];
   free(c);
   return result;
}
```

i=1

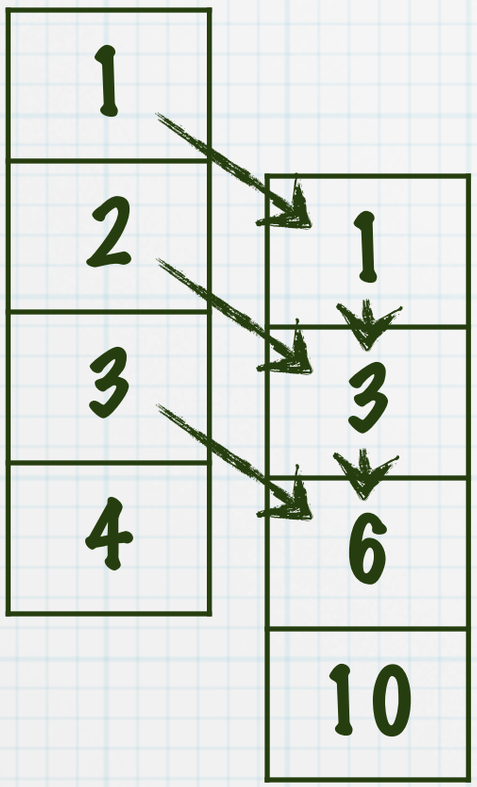
Exemplo




```
int comb(int n, int k) {
   int i, j;
   int maxj = n - k;
   int *c = malloc(sizeof(int) * maxj+1);
   int result = 0;
   for (j = 0; j <= maxj; j++)
      c[j] = j + 1;
   for (i = 1; i < k; i++) {
      for (j = 1; j <= maxj; j++)
         c[j] += c[j - 1];
   }
   result = c[maxj];
   free(c);
   return result;
}
```

i=1

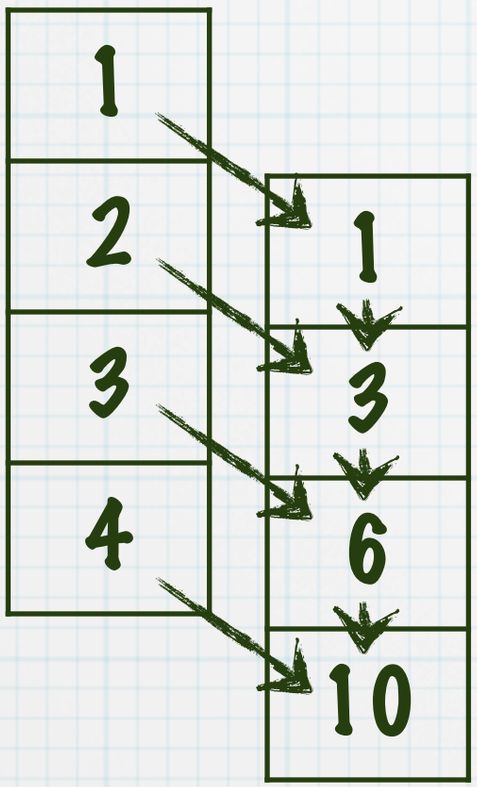
Exemplo




```
int comb(int n, int k) {
   int i, j;
   int maxj = n - k;
   int *c = malloc(sizeof(int) * maxj+1);
   int result = 0;
   for (j = 0; j <= maxj; j++)
      c[j] = j + 1;
   for (i = 1; i < k; i++) {
      for (j = 1; j <= maxj; j++)
         c[j] += c[j - 1];
   }
   result = c[maxj];
   free(c);
   return result;
}
```

i=1

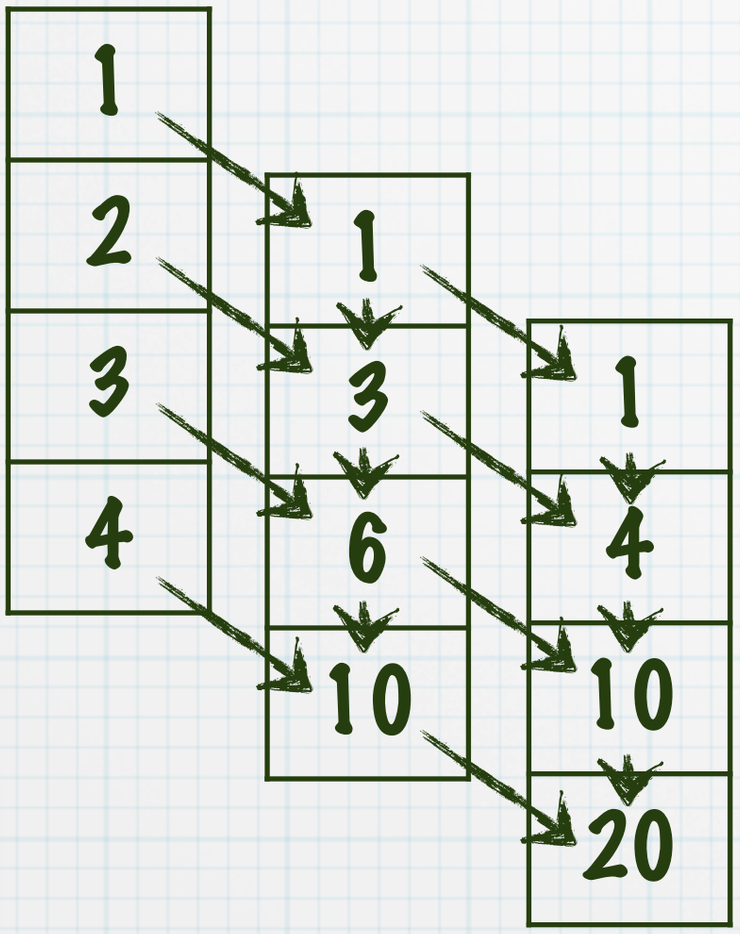
Exemplo




```
int comb(int n, int k) {
   int i, j;
   int maxj = n - k;
   int *c = malloc(sizeof(int) * maxj+1);
   int result = 0;
   for (j = 0; j <= maxj; j++)
      c[j] = j + 1;
   for (i = 1; i < k; i++) {
      for (j = 1; j <= maxj; j++)
         c[j] += c[j - 1];
   }
   result = c[maxj];
   free(c);
   return result;
}
```

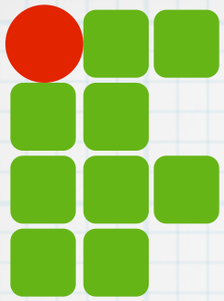
i=1

Exemplo




```
int comb(int n, int k) {
   int i, j;
   int maxj = n - k;
   int *c = malloc(sizeof(int) * maxj+1);
   int result = 0;
   for (j = 0; j <= maxj; j++)
      c[j] = j + 1;
   for (i = 1; i < k; i++) {
      for (j = 1; j <= maxj; j++)
         c[j] += c[j - 1];
   }
   result = c[maxj];
   free(c);
   return result;
}
```

i=2



Fibonacci

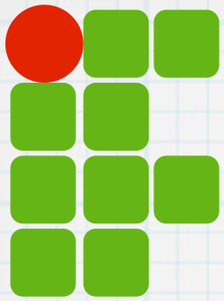
$$\text{fib}(n) = \begin{cases} 1 & \text{Se } n = 1 \\ 1 & \text{Se } n = 0 \\ \text{fib}(n-1) + \text{fib}(n-2) & \text{Se } n > 1 \end{cases}$$

Fibonacci

$$\text{fib}(n) = \begin{cases} 1 & \text{Se } n = 1 \\ 1 & \text{Se } n = 0 \\ \text{fib}(n-1) + \text{fib}(n-2) & \text{Se } n > 1 \end{cases}$$

* Para calcular fib(n)
* Memorizar os dois últimos elementos da sequencia

```
int fib(int n) {
    int a = 1, b = 1, t;
    for (int i = 1; i <= n; i++) {
        t = a;
        a = b;
        b += t;
    }
    return b;
}
```



Conclusão

* problemas de solução recursivas
* Minimizar tempo e espaço
* Induzir uma progressão iterativa de transformações sucessivas

Dúvidas?